SHORTER COMMUNICATIONS

OSCILLATORY BEHAVIOUR IN CONVECTING WATER

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NOMENCLATURE

A,	amplitude of the convective parameter;
D,	cylinder diameter;
g,	gravity field;
h,	cylinder height;
$\mathscr{L}$,	operator;
n,	spatial mode;
Pr,	$= v/k$, Prandtl number;
Ra,	Rayleigh number;
Ra_c,	critical Rayleigh number;
Ra_{osc},	Rayleigh number at oscillations occurrence;
t,	time;
t^*,	characteristic time;
T,	temperature;
u,	velocity perturbation;
z,	vertical coordinate;
β,	coefficient of volume expansion in water around its maximum density;
δ_{th},	thermal boundary layer thickness;
v,	kinematic viscosity;
κ,	thermal diffusivity;
ρ,	fluid density;
θ,	temperature perturbation;
τ,	period of the first temporal mode;
φ,	angle in the horizontal plane.

1. INTRODUCTION

UP TO NOW, a good deal of interest has been generated by the observation of regular temperature oscillations in fluid layers. However, unsteady convective instabilities in enclosures have received little attention; in particular studies in long vertical cylinders are few [1–3].

The existence of such time-dependent instabilities is of some importance in the field of geology, astrophysics and crystal growth.

In this paper, we analyse experiments and observations that we have made on high Rayleigh numbers in completely confined water columns. The investigation of time-dependent effects beyond the onset of convection induced by the density inversion of water was achieved by local temperature measurements.

2. THE EXPERIMENTAL SITUATION

The experimental cell is sketched in Fig. 1. It is a cylinder of variable height h and diameter $D = 3.5$ cm. The top and bottom boundaries are made of copper. The wall consists of 0.2 cm thick Plexiglas. By means of a suitable coolant (alcohol + water) through the copper block, any desired temperature gradient could be maintained in the liquid. In the present case, the bottom is subjected to 0°C and the top to 4°C. Temperature distributions in water were obtained from constantan–manganine thermocouples measurements. These thermocouples were inserted into the fluid at different locations. To obtain temperatures against the z coordinate $T(z)$ and around the mid-height cross-section $T(\varphi)$, thermocouples are equally imbedded in the cylinder wall in direct contact with the fluid. Subsequent experiments showed that their presence did not drastically alter fluid motion.

In order to explore temperature along a diameter, one thermocouple inside an alumina cane was moved laterally. In this case, the presence of the thermocouple affected the nature of temperature measurements because of its interaction with the thermally driven flow.

3. OBSERVATIONS

In certain cell configurations, as the vertical temperature difference is well established and after a time much longer than the thermal relaxation time, the fluid temperature appeared to oscillate spontaneously. A typical plot of local temperature signals is given in Fig. 2a. With constant external conditions, oscillations continued with constant amplitude and frequency over periods as long as 24 h with no noticeable change; but, sometimes they decayed. Under certain conditions convective motion may be either periodic, quasi-periodic or completely erratic.

For all experimental conditions the data are separated into two distinct groups:

$$(1) \qquad \text{for } 0.86 < \frac{h}{D} < 1.14 \quad \text{and} \quad 55 < \frac{Ra_{osc}}{Ra_c} < 73$$

well defined oscillations are established with amplitudes less than 1°C, and periods about 7 mn;

$$(2) \qquad \text{for } 1.14 < \frac{h}{D} < 1.43 \text{ and } 44 < \frac{Ra_{osc}}{Ra_c} < 55$$

random oscillatory convection appeared.

The power spectra were calculated by Fourier transform. Many signals appeared to be composed of a fundamental frequency with higher harmonics superimposed (Fig. 2b). In all the cases, the simplest period is about 7 mn; this corresponds within experimental errors to the thermal diffusion time in the column water estimated to be 6 mn. Consequently, the period is governed by the thermal diffusivity. Generally the frequency of the oscillation was characteristic of the cylinder size and the Rayleigh number; if τ is the period of the first mode τ/Ra is roughly constant $\simeq 28$.
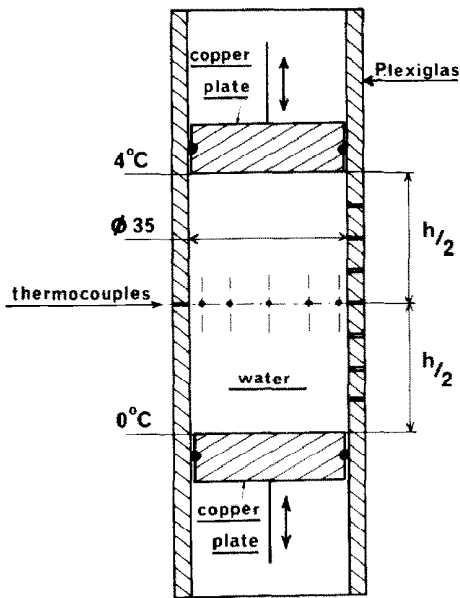
FIG. 1. Experimental cell.

Adding fluoresceine, a heavy salt, oscillations occurred for $h/D = 1.57$ which is higher than in the case of pure water. Amplitudes are less than $1.5°C$ and τ is by and large 13 mn; hence, τ salt $> \tau$ water. This result made evident the role of the density ρ on the onset of convection. The observed oscillations are not near steady convection as the Rayleigh numbers are in a very high range; these Rayleigh numbers being imposed by the geometry. Let us recall that in the region of the maximum density of water the thermal coef-
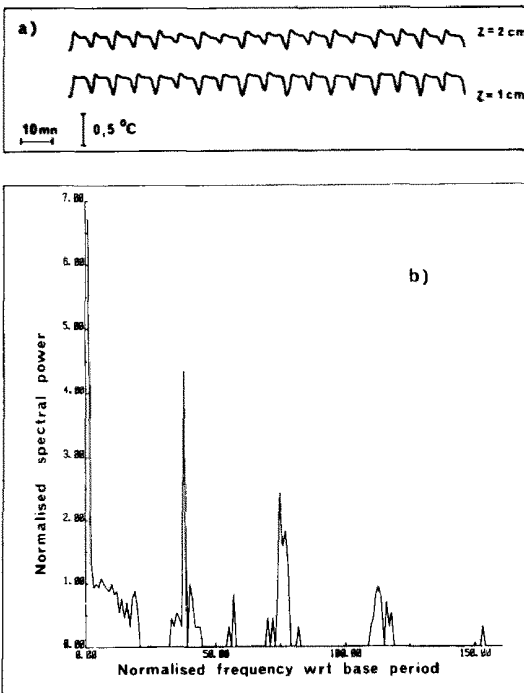

FIG. 2. (a) Typical temperature oscillations recorded by two thermocouples placed on the vertical axis, $h = 3.9$ cm. (b) Spectrum of the signals plotted on Fig. 2a

ficient of expansion approaches zero and the Rayleigh number is normally written becomes inadequate to define the problem. A modified Rayleigh number, then, in a long vertical cylinder filled with water is expressed as:

$$Ra^* = \frac{g\beta(\Delta T)^2 R^4}{vkh}.$$

The critical Rayleigh number Ra_c was determined for every h by a linear stability analysis communicated elsewhere [4].

4. DISCUSSION

Our results differ significantly from previous data in cylinders. Using liquid mercury, $Pr = 0.25$, Verhoeven [2] found that the critical Rayleigh number for the onset of periodic temperature fluctuations Ra_{osc} exceeded the theoretical Ra_c by only 10%. Olson and Rosenberger [3] obtained for monocomponent gases $Ra_{osc}/Ra_c = 5.86$. Whereas for air ($Pr = 0.7$) confined between large horizontal conducting plates in the Rayleigh Bénard problem, Willis and Deardorff [5] obtained $Ra_{osc}/Ra_c = 3.4$ and Krishnamurti [6] $Ra_{osc}/Ra_c = 2.8$. Comparison with our results urges the interesting conclusion that the stability of the fluid is increased not only by the presence of vertical walls but also by using water instead of other fluids.

4.1. Comparison with previous models

Temperature oscillations are associated with unsteady flow convection. In general, there is no obvious correlation between these data and existing theoretical models of convective temperature oscillations, as most of the theoretical models were formulated for the Rayleigh-Bénard geometry. As far as we can discern, none of the simplest models suggested can adequately explain the complex phase of the structure. Howard [7] has proposed a periodically unstable horizontal thermal boundary layer model with possibly release of thermals or plumes. The thickness of the thermal boundary layer δ_{th} increases as $\delta_{th} = \sqrt{\pi\kappa t^*}$, t^* is the necessary time for the boundary layer to be rebuilt after being suddenly broken away. Example: for $h = 4$ cm, the observed period τ is $\simeq 7$ mn. If we calculate the corresponding boundary layer, we will obtain $\delta_{th} = 1.34$ cm which is of the same order of magnitude of the radius $D/2 = 1.7$ cm; whereas Howard assumed that δ_{th} is much lower than the characteristic length of the problem. The hot spot theory is similar to the Welander model [8]; it consists of a hot pocket of fluid which interacts with a pre-existing roll; it goes up very quickly and comes down slowly. There are schematically two main phases in a complete cycle. The first one corresponds to the growing of a hot droplet. The second stage corresponds to the advection of this droplet: after an acceleration and a warming-up, then overturning and destruction of the roll. These unstable motions are associated with thermal anomalies in the fluid that are advected materially around the roll. The models above do not describe the oscillatory behaviour of the present case where the oscillations are in phase along z (Fig. 2a).

Oscillatory convection due to vertical vorticity (v, v) variations or horizontal vorticity (h, v) variations are not available either in this case. A vv variation causes an asymmetric roll to rotate around its own axis, temperature oscillations at a stationary thermocouple will be in phase along the vertical flow direction and out of phase in any radial direction. In hv variation the roll changes its amplitude, contracts during one half-period and finds its initial form again at the end of one period.

Indeed, first there is not a global rotation of the convective structure as at the two diametrically opposite points oscillations are not out of phase contrarily to the results obtained in water when a freezing front goes upwards [9]. Secondly, the profiles of the angular temperature given by four thermocouples located regularly around the cross-section are so different (Fig. 3) that a whole motion of a unique role with the same spatial dependence but modulated by $\sin \omega t$ is excluded.

4.2. Conditions of relaxation oscillations occurrence

Although in many cases the interpretation was ambiguous, the most unsteady phenomena which were observed are represented versus time by a smooth ascension and an abrupt descent; this is characteristic of relaxation oscillations. Likeness with oscillations of relaxation determined theoretically by Busse is striking (Fig. 4). Occurrence of this type of oscillations in fluids with the Prandtl number $Pr \geqslant 1$ is subordinate to two conditions. First the material properties of the fluid layer are not symmetrical with respect to the midplane of the layer. This implies that at the static state only a part of the layer is gravitationally unstable. Secondly, the layer adjacent to the unstable layer has a high heat capacity so that it could serve as a heat reservoir.

Both conditions assume the existence of an inverted bifurcation. The present case satisfies the first condition but not the second one. Indeed, the linear stability analysis [4] has proven that a stagnant layer of water exists near 4°C when the asymmetry of the density ρ with respect to the mid-plane of the layer resulting from the law ρ (T) is taken into account. However, for aspect ratios near the unity, calculations have shown that the thickness of the non-convective water layer does not exceed 0.3 cm and thus it cannot play the role of a heat reservoir.

The physical mechanism of a periodic relaxation process can be summarized as follows; as the convective motion grows, it flattens the temperature gradients until the effective Rayleigh number is lowered so far that the convection dies away. As soon as the heat transport by conduction has increased the temperature gradients, the convective layer is rebuilt and convection will start growing again.

For aspect ratio near unity, the duration of the establishment phase of temperature is $t^* \simeq 6s$. This time is too short to allow the Rayleigh number to decrease beyond its critical value. For this reason, the solution given above must be eliminated and two other processes admitting different response times remain to be defined. One can note that the observed oscillations occur in pure water for h/D between 0.86 and 1.2. Now, if one examines the stability curves in water [4], one will notice that for $h/D = 0.75$ the convective modes $n = 1$ and $n = 0$ have crossed each other, thus it will be possible to jump from one to the other. Nevertheless, oscillations between these two modes can only exist if an inverted bifurcation exists.

4.3. Subcritical instabilities in water between 0 and 4°C

Veronis [10] has demonstrated in a water layer of infinite lateral extent case "free – free" that any asymmetry in the horizontal plane entails an inverted bifurcation and so a
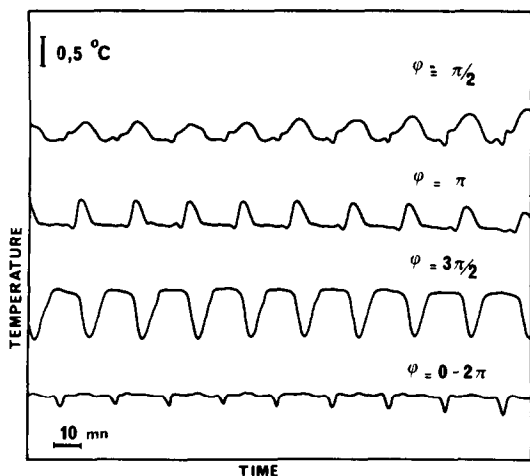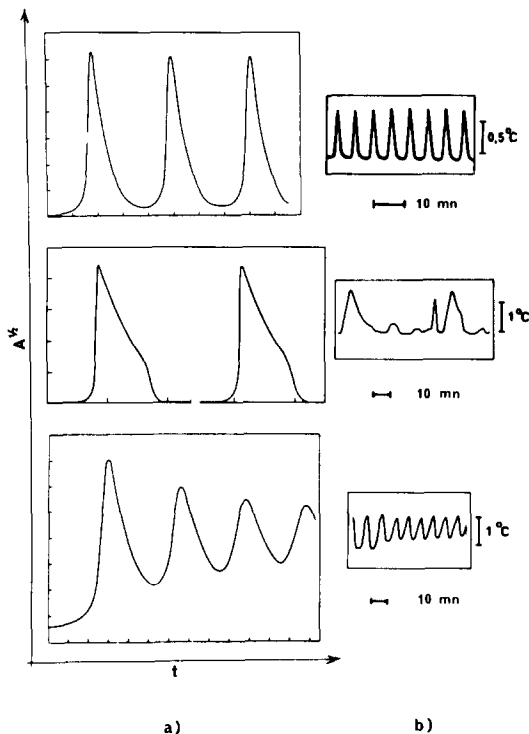

FIG. 4. (a) Relaxation oscillations after Busse. [(J. Fluid Mech. 30, 625 (1967)]. (b) Recordings of temperature in the course of time along z for $h/D \simeq 1$.

FIG. 3. Angular temperatures given by four thermocouples regularly placed at the mid-cross-section.

subcritical Rayleigh number. He has developed the Rayleigh number, the fluctuations of temperature and velocity in a series such as:

$$u = \varepsilon u_0 + \varepsilon^2 u_1 + \varepsilon^3 u_2,$$

$$\theta = \varepsilon \theta_0 + \varepsilon^2 \theta_1 + \varepsilon^3 \theta_2,$$

$$Ra = Ra_0 + \varepsilon Ra_1 + \varepsilon^2 Ra_2 + ...,$$ (1)

Ra_0 being the critical Rayleigh number Ra_c given by the linear theory. Then, he solved the governing equations up to the second order, which led to the following conclusions: (a) when convective cells are hexagonal a horizontal asymmetry is introduced and Ra_1 is different from zero. Then $Ra - Ra_0 = \varepsilon Ra_1$. If $Ra_1 \gtrless 0$, $\varepsilon \gtrless 0$ requires that $Ra < Ra_0$. Consequently, a finite amplitude instability is possible for a subcritical Rayleigh number; (b) when convective cells occur as 2 rolls, $Ra_1 = 0$ and $Ra_2 < 0$. Symmetry in the horizontal plane means that only the terms in ε^{2p} persist in the expansion of Ra. Hence, in this case, any subcritical instability is only possible if Ra_2 is negative. In fact starting from (1) one can write:

$$\varepsilon^2 = \frac{Ra - Ra_0}{Ra_2}.$$

As ε^2 is > 0, Ra is $< Ra_0$ when $Ra_2 < 0$.

We shall follow the same argument for the modes $n = 0$ and $n = 1$. Already, we notice a similarity between hexagonal flow and the axisymmetrical mode $n = 0$ which leads us to imagine that in the mode $n = 0$ an inverted bifurcation may exist. But in the mode $n = 1$ where there is a symmetry in the horizontal plane it is probably impossible. Let us consider the linear problem defined by an operator $\mathscr{L}$, with $\mathscr{L}\theta_1 = 0$ and the non-linear problem where the quadratic terms of θ and u are considered. The non-linear terms are,

$$N[\theta_1 u_1, u_1^2, \theta_1^2], \ \mathscr{L}\theta_2 = Ra_1 \ \theta_1 + \frac{\partial \theta_1}{\partial z} u_1^2. \qquad (2)$$

Ra_1 is such that the second member of (2) is orthogonal to the core of $\mathscr{L}$. That yields:

$$Ra_1 \langle \theta_1, \theta_1 \rangle = \left\langle \theta_1 \frac{\partial \theta_1}{\partial z} u_1^2 \right\rangle. \qquad (3)$$

The brackets denote the integral over the volume and the subscripts 1 and 2 indicate the order of the development. The integral (3) can be factorized into an integral of z and an integral in the horizontal plane. The two integrals must be jointly null so that an instability may exist.

As $\theta(\varphi)$ and $u(\varphi)$ are expressed in $\cos n\varphi$; for $n = 0$, $\cos n\varphi$ $= 1 \Rightarrow Ra_1 \neq 0 \Rightarrow$ subcritical instability with an inverted bifurcation.

For $n = 1, \int_0^{2\pi} \cos^3 \varphi \, d\varphi = 0 \Rightarrow Ra_1 = 0 \Rightarrow$ critical instability with a normal bifurcation.

As a rule, to define a bifurcation scheme of these two modes and the possibility of relaxation oscillations it would be necessary to have the amplitudes of the convective velocities in order to deduce their contribution to the heat flux and to know which of them absorbed most heat. However we are in a position to give a diagram illustrating the amplitude A of temperature or velocity fluctuations versus the Rayleigh number Ra (Fig. 5). As the mode $n = 1$ is the more unstable, $Ra_{c(1)}$ is lower than $Ra_{c(0)}$. Basing our investigation upon this diagram, we can imagine that the mode $n = 1$ changes its amplitude A_1 contracting and the mode $n = 0$ could grow to diminish afterwards thus permitting a periodic exchange between $n = 0$ and $n = 1$.

5. CONCLUSION

To answer some of the questions generated by the different convective states observed (periodic, quasi-periodic, erratic) the physical mechanism suggested as responsible of oscillatory flows in a small confined cylinder has been based on the conditions of oscillations occurrence and the nature of the flow. It consists of relaxation oscillations between the axisymmetrical mode $n = 0$ and the first diametral antisymmetrical mode $n = 1$.

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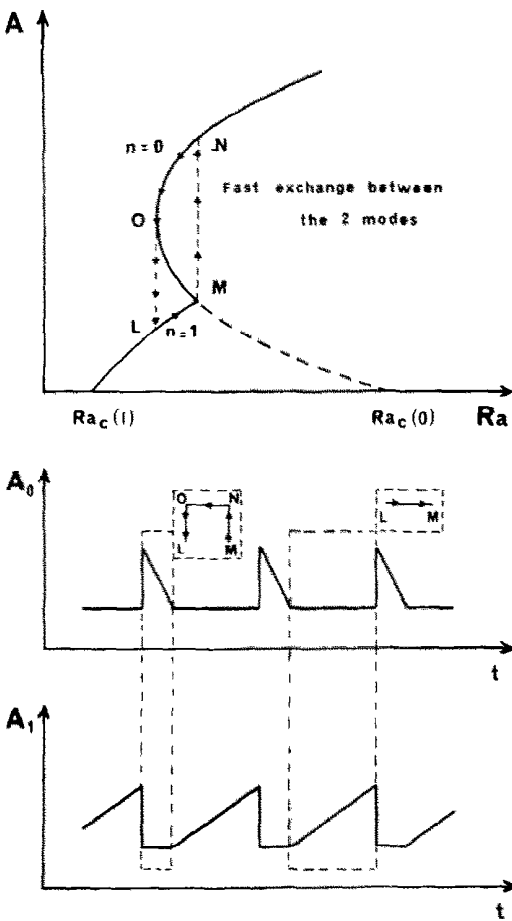10. G. Veronis, Penetrative convection, *Astrophys. J.* **137**, 641–663 (1963).

FIG. 5. Mechanism of relaxation oscillations between $n = 0$ and $n = 1$.